2007 International Nuclear Atlantic Conference - INAC 2007
Santos, SP, Brazil, September 30 to October 5, 2007
ASSOCIAÇÃO BRASILEIRA DE ENERGIA NUCLEAR - ABEN
ISBN: 978-85-99141-02-1

THE CELLULAR PARTICLE SWARM OPTIMIZATION ALGORITHM

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ABSTRACT

This work presents a variant of the Particle Swarm Optimization (PSO) original algorithm, the Cellular-PSO. Inspired by the cellular Genetic Algorithm (GA), particles in Cellular-PSO are arranged into a matrix of cells interconnected according to a given topology. Such topology defines particle's neighborhood, inside which social adaptation may occur. As a consequence, population diversity is increased and the optimization process becomes more efficient and robust. The proposed Cellular-PSO has been applied to the nuclear reactor core design optimization problem and comparative experiments demonstrated that it is superior to the standard PSO.

1. INTRODUCTION

In a nuclear reactor design, a wide variety of optimization problems arise from the need of achieving high efficiency, availability and safety levels. Optimum design parameters are searched in very complex search spaces, under lots of constraints. Therefore, efficiency and robustness of the optimization techniques are quite important.

Several decades ago, traditional gradient-based optimization techniques had been used [1]. Due to the non-linearity, multimodality and, poor knowledge about the search domain of most complex problems, the use of more robust and appropriate techniques such as Simulated Annealing (SA) [2] and Genetic Algorithms (GA) [3-7] have been proposed.

Seeking for enhancements in the optimization processes, other methodologies have been proposed. Among them, Particle Swarm Optimization (PSO) [8], a population-based metaheuristic has demonstrating some advantages over other techniques. Several applications of PSO to nuclear problems have been recently published in literature [9-12]. In this work, a variant of the standard PSO - the Cellular-PSO - is described and results of its application to a nuclear core design optimization problem demonstrated advantages over the standard PSO.

2. THE CELLULAR PSO

Particle Swarm Optimization (PSO) is an optimization metaheuristic inspired by the behavior of biological swarms and social adaptation. In PSO, a swarm of structures encoding solution candidates ("particles") "fly" in the n-dimensional search space of the optimization problem looking for optima or near-optima regions. The *position* of a particle represents a solution candidate itself, while the *velocity* attribute, provides information about direction and changing rate. Particles are guided by two components: i) cognitive information based on particles' own experience and ii) social information based on observation of neighbors. Let $\overrightarrow{X}_i(t) = \{x_{i,1}(t),...,x_{i,n}(t)\}$ and $\overrightarrow{V}_i(t) = \{v_{i,1}(t),...,v_{i,n}(t)\}$ be, respectively, the position and the velocity of particle i in time t, in an n-dimensional search space. Considering that $\overrightarrow{pBest}_i(t) = \{pBest_{i,1}(t),..., pBest_{i,n}(t)\}$ is the best position already found by particle i until time t and $\overrightarrow{gBest}_i(t) = \{gBest_{i,1}(t),..., gBest_{i,n}(t)\}$ is the best position already found by a neighbor until t, the PSO updating rules for velocity and position are given by:

$$v_{i,n}(t+1) = w.v_{i,n}(t) + c_1.r_1.(pBest_{i,n}(t) - x_{i,n}(t)) + c_2.r_2.(gBest_{i,n}(t) - x_{i,n}(t)) \quad (1)$$

$$x_{i,n}(t+1) = x_{i,n}(t) + v_{i,n}(t+1) \quad (2)$$

where r_1 and r_2 are random numbers between 0 and 1. Coefficients c_1 and c_2 are given acceleration constants towards $\overrightarrow{pBest}$ and $\overrightarrow{gBest}$ respectively and w is the inertia weight.

The inertia weight, w, is the responsible for the scope of the exploration of the search space. High values of w promote global exploration and exploitation, while low values, lead to local search. A common approach to provide balance between global and local search is to linearly decrease w during the search process.

The swarm is randomly initialized. Then, while stopping criterion is not reached, particles move according velocity and positions equations (eqs. 1 and 2). The PSO algorithm pseudo code can be seen in Figure 1.

```
Algorithm PSO
begin
   for i=1 to n_particles do begin
      randomize(Xi); randomize(Vi);
   end;
   for iter=1 to itermax do begin
      for i=1 to n_particles do evaluate (Xi);
      for i=1 to n_particles do update(pBesti,gBest);
      for i=1 to n_particles do begin
         Vi = w*Vi+c1*r1*(pBesti-Xi)+c2*r2*(gBest-Xi);
         Xi = Xi+Vi;
      end;
   end;
end.
```

Figure 1. Standard PSO pseudo code.

The proposed Cellular-PSO is a parallel/distributed approach in which particles are arranged in a 2D-grid, as illustrated in Figure 2. In the Cellular-PSO, $\overrightarrow{gBest}$ is no more visible to all particles. Instead, particles use a $\overrightarrow{lBest}$ (local best), which is the best particle among the neighbors. Such neighborhood restriction delays the information exchange between non-neighbor particles, increasing the diversity in the search. As consequence, the method becomes more robust and efficient.

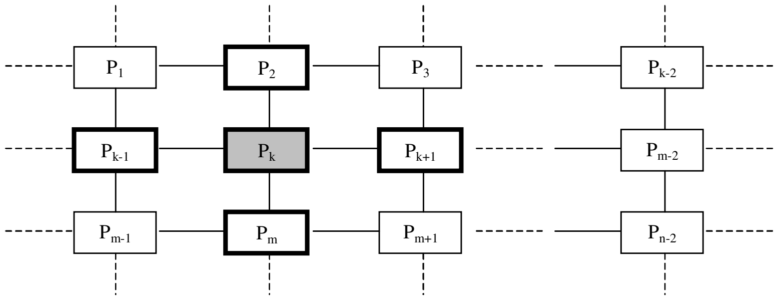

Figure 2. Cellular-PSO

3. THE OPTIMIZATION PROBLEM

It is considered a simplified cylindrical three-enrichment-zone Pressurized Water Reactor (PWR), with typical cell composed by moderator (light water), cladding and fuel. Figure 3 illustrates such reactor. The design parameters as well as their ranges are shown in Table 1.

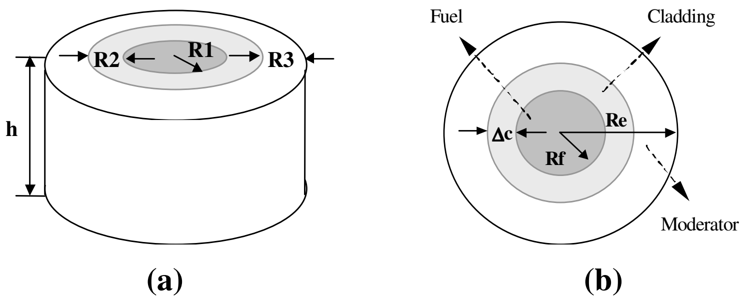

Figure 3. (a) The reactor and (b) it's typical cell

Table 1. Optimization parameters range

Parameter	*Symbol*	*Range*
Fuel radius (cm)	Rf	0.508–1.27
Cladding thickness (cm)	Δc	0.0254–0.254
Moderator thickness (cm)	Re	0.0254–0.762
Enrichment of zone 1 (%)	E_1	2.0–6.0
Enrichment of zone 2 (%)	E_2	2.0–6.0
Enrichment of zone 3 (%)	E_3	2.0–6.0
Fuel material	M_f	{U-metal or UO2}
Cladding material	M_c	{zircaloy-2, aluminum or stainless-304}

The objective of the optimization problem is to maximize the average thermal flux, ϕ_{AVE}, of the proposed reactor, considering as constraints the criticality, the sub-moderation and a maximum peak factor fp_{MAX}. The optimization problem can be written as follow:

Minimize:

$$\phi_{AVE}\,(R_f, \Delta_c, R_e, E_1, E_2, E_3, M_f, M_c)$$

Subject to:

$$fp(R_f, \Delta_c, R_e, E_1, E_2, E_3, M_f, M_c) \leq fp_{MAX} \tag{3}$$

$$0.99 \le k_{eff}(R_f, \Delta_c, R_e, E_1, E_2, E_3, M_f, M_c) \le 1.01 \quad (4)$$
$$dk_{eff}/dV_m > 0 \quad (5)$$
$$R_{f\,min} \le R_f \le R_{f\,max} \quad (6)$$
$$\Delta_{c\,min} \le \Delta_c \le \Delta_{c\,max} \quad (7)$$
$$R_{e\,min} \le R_e \le R_{e\,max} \quad (8)$$
$$E_{1\,min} \le E_1 \le E_{1\,max} \quad (9)$$
$$E_{2\,min} \le E_2 \le E_{2\,max} \quad (10)$$
$$E_{3\,min} \le E_3 \le E_{3\,max} \quad (11)$$
$$M_f = \{UO_2, \text{ or U-metal}\} \quad (12)$$
$$M_c = \{\text{Zircaloy-2, Aluminum or Stainless-304}\} \quad (13)$$

where V_m is the moderator volume. The *Hammer* system [13] was used to make the reactor physics calculations. The fitness function to be minimized was developed in such a way that, if all constraints are satisfied, it assumes the value of the average thermal flux, ϕ_{AVE}. Otherwise, it is penalized proportionally to the disagreement on the constraint. The expert, according to the requirements and the priorities of the problem should set up the penalization constants r_i.

$$f = \begin{cases} \phi_{AVE}, & \Delta k_{eff} \le 0.01;\ fp \le 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} > 0 \\ \phi_{AVE} - r_1.\Delta k_{eff}, & \Delta k_{eff} > 0.01;\ fp \le 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} > 0 \\ \phi_{AVE} - r_2.\Delta fp, & \Delta k_{eff} \le 0.01;\ fp > 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} > 0 \\ \phi_{AVE} - r_3.\dfrac{\Delta' k_{eff}}{\Delta Vm}, & \Delta k_{eff} \le 0.01;\ fp \le 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} < 0 \\ \phi_{AVE} - r_1.\Delta k_{eff} - r_2.\Delta fp, & \Delta k_{eff} > 0.01;\ fp > 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} > 0 \\ \phi_{AVE} - r_1.\Delta k_{eff} - r_3.\dfrac{\Delta' k_{eff}}{\Delta Vm}, & \Delta k_{eff} > 0.01;\ fp \le 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} < 0 \\ \phi_{AVE} - r_2.\Delta fp - r_3.\dfrac{\Delta' k_{eff}}{\Delta Vm}, & \Delta k_{eff} \le 0.01;\ fp > 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} < 0 \\ \phi_{AVE} - r_1.\Delta k_{eff} - r_2.\Delta fp - r_3.\dfrac{\Delta' k_{eff}}{\Delta Vm}, & \Delta k_{eff} > 0.01;\ fp > 1.40;\ \dfrac{\Delta' k_{eff}}{\Delta Vm} < 0 \end{cases} \quad (14)$$

Where, $\Delta k_{eff} = |1.0 - k_{eff}|$; $\Delta fp = fp - 1.40$; $\Delta Vm = 0.03Vm$

4. EXPERIMENTS AND RESULTS

In order to evaluate the efficiency and robustness of the Cellular-PSO, it has been submitted to 10 experiments (see Table 2), with different random seeds. Results are compared to those obtained by a standard PSO (Table 3). For both, standard and cellular PSO, population size was 20. Constant c_1 and c_2 have been set to 2.0. The inertia weight, w, decreased linearly from 0.8 to 0.2.

Table 2. Results obtained by the Cellular PSO

Exp.	*Fitness*	*Flux*[3]	*Keff*	*fp*	*Rf*	*ΔC*	*ΔM*[2]	*E1*	*E2*	*E3*	*Mf*[1]	*Mc*[1]
1	1.53E-04	1.53E-04	0.9901	1.3990	0.2241	0.4241	0.7716	2.6786	4.6784	4.6086	100	200
2	1.62E-04	1.62E-04	0.9900	1.3999	0.2001	0.4001	0.7019	3.2753	3.5870	5.8308	100	200
3	1.62E-04	1.62E-04	0.9900	1.3991	0.2000	0.4000	0.7019	3.2757	3.5893	5.8405	100	200
4	1.50E-04	1.50E-04	0.9900	1.3971	0.2779	0.4766	0.8578	2.7423	3.0601	4.8282	100	200
5	1.60E-04	1.60E-04	0.9905	1.3842	0.2022	0.4011	0.7022	3.2109	3.5474	5.8764	100	200
6	1.56E-04	1.56E-04	0.9900	1.3989	0.2349	0.4349	0.7710	2.9883	3.2723	5.2744	100	200
7	1.55E-04	1.55E-04	0.9901	1.3987	0.2004	0.4000	0.7166	2.7461	5.0205	4.7759	100	200
8	1.58E-04	1.58E-04	0.9901	1.3999	0.2195	0.4195	0.7383	3.0852	3.3774	5.4656	100	200
9	1.57E-04	1.57E-04	0.9918	1.3922	0.2019	0.4014	0.6900	3.0158	3.7378	5.3174	100	202
10	1.60E-04	1.60E-04	0.9909	1.3892	0.2001	0.4000	0.6973	3.2189	3.5780	5.8745	100	200
Ave	***1.574E-04***											
SDv	***3.599E-06***											

[1] *Material codes: U-metal=100;* UO_2*=101; Al=200; Stainless-304=201; Zircaloy-2=202*
[2] *ΔM=Re-Rf-ΔC* [3]*source normalized*

Table 3. Results obtained by the Standard PSO

Exp.	*Fitness*	*Flux*[3]	*Keff*	*fp*	*Rf*	*ΔC*	*ΔM*[2]	*E1*	*E2*	*E3*	*Mf*[1]	*Mc*[1]
1	1.182E-04	1.182E-04	0.9976	1.3982	0.2189	0.2795	0.2795	3.8723	4.1539	5.9535	100	201
2	1.523E-04	1.523E-04	0.9900	1.3999	0.2364	0.4363	0.7926	2.6949	4.2611	4.7230	100	200
3	1.552E-04	1.552E-04	0.9900	1.4000	0.2186	0.4186	0.7118	2.9257	3.1901	5.0189	100	202
4	1.590E-04	1.590E-04	0.9900	1.4000	0.2030	0.4030	0.6873	3.0926	3.3731	5.3065	100	202
5	1.219E-04	1.219E-04	0.9979	1.4000	0.2003	0.2178	0.5777	2.9621	3.1817	4.4726	100	201
6	1.165E-04	1.165E-04	0.9988	1.3988	0.2709	0.3366	0.7951	3.5480	3.8188	5.5293	100	201
7	1.482E-04	1.482E-04	0.9900	1.4000	0.2535	0.4535	0.8052	2.4989	3.9098	4.1383	100	202
8	1.614E-04	1.614E-04	0.9900	1.3999	0.2023	0.4022	0.7063	3.2545	3.5636	5.7894	100	200
9	1.163E-04	1.163E-04	0.9900	1.4000	0.2848	0.3567	0.8251	3.4489	3.7166	5.3198	100	201
10	1.187E-04	1.187E-04	0.9902	1.3998	0.3153	0.7607	0.7607	2.9929	3.2206	4.5678	100	201
Ave	***1.368E-04***											
SDv	***1.879E-05***											

[1] *Material codes: U-metal=100;* UO_2*=101; Al=200; Stainless-304=201; Zircaloy-2=202*
[2] *ΔM=Re-Rf-ΔC* [3]*source normalized*

Observing the average fitness and standard deviation it can be noted that Cellular-PSO demonstrates to be more efficient and robust than the standard PSO. Low values of fitness in experiments 1, 5, 6, 9 and 10 show the susceptibility of the standard PSO to be trapped into local optima. Note that on such experiments the same combination of materials (U-metal and Stainless-304) have been found. Deeper investigations were made fixing the materials obtained in experiments 1 to 10. In such investigations, the best results (fitness) found were *f=1.62E-04* for U-metal and Aluminum, *f=1.59E-04* for U-metal and Zircaloy-2, and *f=1.23E-04* for U-metal and Stainless-304. In fact, the low values obtained by standard PSO in experiments 1, 5, 6, 9 and 10 correspond to near-optimum solutions if U-metal and Stainless-304 are fixed. Cellular-PSO has not been trapped into such local optimum region.

3. CONCLUSIONS

In this work, the Cellular PSO demonstrated to be more efficient and robust than the standard PSO, due to the neighborhood strategy used. Other strategies can be find in literature, however, the cellular model is faster, easier to implement and parallelism is more natural. Future works include the parallel implementation of the Cellular-PSO and investigation of enhancements in the model, in order to improve efficiency, robustness and speed.

ACKNOWLEDGMENTS

Cláudio M. N. A. Pereira and Roberto Schirru are supported by CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico).

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